

# Evolution of Bulk Scale Factor in Warped Space-time

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## Abstract

In this work the role of extra dimensions in the accelerated universe through the scenario of higher-dimensional Friedmann-Robertson-Walker (FRW) cosmology has been studied. For this purpose, we first consider warped space-time in the standard flat brane scenario as the modified form of Robertson-Walker (RW) metric in five-dimension (5D) space-time and then the variation of the *bulk scale factor* (warp factor), with respect to both space-like and time-like extra dimensions is obtained. Finally, it is shown that both of two types of extra dimensions are important in this scenario and also the *bulk scale factor* plays two different roles.

Keywords: Extra dimensions, Warped space-time, Brane-world Scenario, Accelerated Universe.  
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## 1 Introduction

One of the most interesting aspects of modern cosmology concerns the acceleration of the cosmological expansion. Recent data coming from the supernova type Ia [1] and also cosmic microwave background radiation [2] observations, strongly indicate that the expansion of the universe is accelerating, contrary to previous expectations. Most attempts to explain this acceleration involve the introduction of dark energy, as a source of the Einstein field equations. The nature of the dark energy is unknown but it behaves like a fluid with a large negative pressure and one possible candidate for the dark energy might be a very small cosmological constant in Einstein equation.

Another, possibly related problem, involves the existence of dark matter where it is needed in the rotation curves of spiral galaxies where the galactic rotational velocities differ from the velocities that predicted by the Newtonian gravitational potentials due to the luminous matter in the galaxies, in other words observations of spiral galaxies, elliptical galaxies and galactic clusters indicates that these objects contain a large amount of dark matter.

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We recognize as the standard cosmological model the  $\Lambda$ CDM model, which describes standard gravity with the scalar curvature  $R$ , with the inclusion of the cosmological constant  $\Lambda$  and the nonrelativistic dustlike matter. The presence of dark energy may be considered with diverse modifications of this standard scenario. The two main approaches are the quintessence way [3], in which one considers the possibility of changing the standard scenario with the inclusion of scalar fields, and another one is the  $f(R)$ -theories of gravity [4], in which the effect of some functions of scalar curvature has been considered.

Many of cosmologists have suggested that the extra dimensions may play the major role in acceleration of the universe [5]. It is predicted that the fifth dimension produces a new dynamic force in 4D space-time [6]. Therefore, in order to explain the acceleration of the universe in the higher dimensions, some researchers have considered the time dependent size of the extra dimension as a suitable candidate [7].

Brane scenario is being intensively investigated as a sound candidate for solving the hierarchy problem and also it appears in other fundamental problems in high energy physics [8, 9]. The brane concept which was first introduced in [10], is formulated in terms of a single infinite extra dimension according to which the physical world appears as a 4D space-time embedded into an anti-de Sitter (AdS) space-time of one higher spatial dimension. This scenario starts with the 5D space-time with warped geometry described by a single function which is real and depends only on the fifth or bulk coordinate where this coordinate is considered as an infinite space-like coordinate. The scenario is admissible under the presence of scalar fields which are also supposed to depend only on the extra dimension and work to stabilize the brane-world configuration. Let us name this as the standard brane-world scenario, since it is governed by some Einstein-Hilbert action which includes gravity and scalar fields in a standard way [11]. Other ideas of modifying the scenario of [10] have been studied in [12, 13, 14, 15], that include the change of the scalar field dynamics to the tachyonic dynamics [12] or the inclusion of a Gauss-Bonnet term [13]. Also, some authors consider extensions of torsion scalar  $T$ , gravity to  $f(T)$  theories [16].

The standard flat brane-world scenario is described by real scalar field  $\phi$  interacting with gravity via the usual Einstein-Hilbert action, which has the general form as follows [10],

$$S = \int d^4x dy \sqrt{g} \left( -\frac{1}{4} R + L \right). \quad (1)$$

We have used the standard notation and  $4\pi G = 1$ . The scalar field  $\phi$  is governed by standard dynamics with the following Lagrange density

$$L = \frac{1}{2} g_{ab} \partial^a \phi \partial^b \phi - V(\phi). \quad (2)$$

where  $g_{ab}$  describes the 5D space-time with  $a, b = 0, 1, 2, 3, 4$  and  $x_4 \equiv y$  stands for the extra dimension.

The line element is given by [17]

$$\begin{aligned} ds^2 &= g_{ab} dx^a dx^b = a^2(y) \eta_{\mu\nu} dx^\mu dx^\nu + \epsilon^2 dy^2, \\ g_{ab} &= \text{diag}(a^2(y), -a^2(y), -a^2(y), -a^2(y), \epsilon^2), \end{aligned} \quad (3)$$

where  $a^2(y)$  is the warp factor and as mentioned it is supposed to be a real function of the extra dimension which gives rise to the warped geometry and  $\eta_{\mu\nu}$  describes the 4D flat space-time, with  $\mu, \nu = 0, 1, 2, 3$ . We use  $\epsilon^2 = -1$  for the Space Like Extra Dimensions (SLED) and  $\epsilon^2 = +1$  for the Time Like Extra Dimensions (TLED). The 5D metric given above, implies that the scalar of curvature, the scalar field and the warp only depend on the fifth coordinate  $y$ , i.e.  $R \equiv R(y)$ ,  $\phi \equiv \phi(y)$  and  $a \equiv a(y)$  respectively. We name this the standard flat brane scenario.

Recently, we proposed a higher dimensional cosmology based on FRW model and brane-world scenario [18]. We considered the warp factor in the metric (3) as a scale factor, which we called as *bulk scale factor*. In the previous work Einstein equation in 5D space-time has been solved and the evolution of *bulk scale factor* with space-like and time-like extra dimensions has been considered as well. We also showed that extra dimensions can produce the negative pressure in the slow roll regime in the early 5D universe [19]. In this work we use the action (1) and metric (3), but we persist that contrary to usual brane-world scenarios that consider the SLED, we also consider the TLED. Therefore, we consider two shapes of the 5D energy-momentum tensor for SLED and TLED separately. The energy-momentum tensor for SLED reads as,

$$T_b^a = \text{diag}(\rho, -P, -P, -P, -\tilde{P}), \quad (4)$$

and for TLED is,

$$T_b^a = \text{diag}(\rho, -P, -P, -P, \tilde{\rho}). \quad (5)$$

Because of the homogeneity condition, we consider,  $\tilde{P} = P$  and  $\tilde{\rho} = \rho$ , as the pressure and density of the fifth component of perfect fluid energy-momentum tensor.

The outline of paper is as follows: In Sect.2, the solution of Einstein equations in the standard RW metric is recalled briefly. In Sect.3, we solve the Einstein equations for the warped geometry- as the generalized 5D RW metric in the brane cosmology scenario- and obtain the evolution of the *bulk scale factor* respect to SLED and TLED. In Sect.4, we discuss the accelerated expansion in this scenario. Some discussions and conclusions are given in the final section.

## 2 Evolution of the Scale factor for the standard 4D flat space-time

In comoving coordinates, the cosmological space-time is given by [20],

$$ds^2 = dt^2 - a^2(t)[dx_1^2 + dx_2^2 + dx_3^2], \quad (6)$$

where  $a(t)$  is called the scale factor.

At cosmic scale, the contents of the universe look like non-interacting tiny gas particles having homogeneous and isotropic distribution. So, the cosmic matter behaves like a perfect fluid and its energy-momentum tensor is given by,

$$T_\nu^\mu = \text{diag}(\rho, -P, -P, -P), \quad (7)$$

and the equation of state for this fluid is,

$$P = \omega\rho, \quad (8)$$

where  $\rho$  and  $P$  are energy and pressure respectively.

On the other hand, Einstein's field equations are given by,

$$R_{\mu\nu} = -8\pi G(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T). \quad (9)$$

Thus, for non-vanishing components of  $R_{\mu\nu}$  and  $T_{\mu\nu}$ , we have from (9),

$$(\frac{\dot{a}}{a})^2 = \frac{8\pi G}{3}\rho, \quad (10)$$

where dot denotes derivative with respect to time.

The Bianchi identities are given by,

$$\nabla_\nu T^{\mu\nu} = 0, \quad (11)$$

which can be rewritten as,

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^\nu}(\sqrt{-g}T^{\mu\nu}) + \Gamma_{\alpha\nu}^\mu T^{\alpha\nu} = 0. \quad (12)$$

In the RW space-time, for non-vanishing components of  $\Gamma_{\alpha\nu}^\mu$  and  $T^{\alpha\nu}$ , we have,

$$\dot{\rho} + 3H(\rho + P) = 0. \quad (13)$$

This is the conservation equation for the homogeneous perfect fluid constituting the cosmic fluid. It is just enough to choose the set of Eqs. (10) and (13) for our work in cosmic evolution in FRW cosmology. Thus, from  $P = \omega\rho$ , the conservation of energy equation (13) becomes,

$$\frac{\dot{\rho}}{\rho} = -3(1 + \omega)\frac{\dot{a}}{a}. \quad (14)$$

This can be integrated to obtain,

$$\rho \approx a^{-3(1+\omega)}. \quad (15)$$

We then find from (10) and (15), the expansion of the universe goes as [21],

$$a \approx t^{\frac{2}{3(1+\omega)}}. \quad (16)$$

In particular, for dark energy fluid with  $-1 < \omega < -\frac{1}{3}$ , we obtain

$$\rho \approx a^m, \quad (17)$$

and

$$a \approx t^n. \quad (18)$$

where  $-2 < m < 0$  and  $1 < n < \infty$ . In this case, the energy density of the universe decrease and the acceleration of the universe increase ( $a > 0$ ). And also for  $\omega = -1$  or vacuum, we have

$$\rho \approx a^0 = cte. \quad (19)$$

In this case the energy density is independent of  $a$ , but we have inflation for the scale factor as,

$$a \approx t^\infty. \quad (20)$$

Thus, in any case, the scale factor vanishes if  $t \rightarrow 0$  and the density at that time becomes infinite. This point is known *Big Bang*.

### 3 Evolution of the *bulk Scale factor* for the 5D warped space-time

Similarly, we solve Einstein's field equations in the 5D space-time (3),

$$R_{ab} = -8\pi G(T_{ab} - \frac{1}{2}g_{ab}T). \quad (21)$$

Thus, for non-vanishing components of  $R_{ab}$  and  $T_{ab}$ , we obtain from (21) for  $ab = 44$ , in SLED case as,

$$\frac{\tilde{a}''}{\tilde{a}} = \pi G(2P - \rho). \quad (22)$$

where  $\tilde{a} \equiv a(y)$  is *bulk scale factor*, and prime denotes derivative with respect to  $y$  as a SLED. Also for  $ab = (11, 22, 33)$ , we obtain from (21),

$$\frac{3\tilde{a}'^2}{\tilde{a}^2} + \frac{\tilde{a}''}{\tilde{a}} = -4\pi G\rho. \quad (23)$$

With connecting Eqs.(22) and (23), we obtain,

$$\frac{\tilde{a}'^2}{\tilde{a}^2} = \frac{8\pi G}{3}(\frac{-3\rho}{8} - \frac{P}{4}). \quad (24)$$

On the other hand, similar to SLED case, for TLED we obtain,

$$\frac{\ddot{\tilde{a}}}{\tilde{a}} = -3\pi GP, \quad (25)$$

and

$$\frac{3\dot{\tilde{a}}^2}{\tilde{a}^2} + \frac{\ddot{\tilde{a}}}{\tilde{a}} = 4\pi G(2\rho - P). \quad (26)$$

where dot denotes derivative with respect to  $y$  as a TLED. with connecting Eqs. (25) and (26), we obtain,

$$\frac{\dot{\tilde{a}}^2}{\tilde{a}^2} = \frac{8\pi G}{3}(\frac{P}{8} - \rho). \quad (27)$$

Also, from the Bianchi identities,  $\nabla_b T^{ab} = 0$ , we obtain the conservation equation for SLED as,

$$P' + \frac{\tilde{a}'}{\tilde{a}}(\rho + P) = 0, \quad (28)$$

and for TLED,

$$\dot{\rho} + 3\frac{\dot{\tilde{a}}}{\tilde{a}}(\rho + P) = 0. \quad (29)$$

We would like give the *bulk scale factor*  $\tilde{a}$  respect to variation of  $y$  in the bulk. Therefore, if we consider  $P = \omega\rho$ , with integration of (28) for SLED, the conservation of energy equation becomes,

$$\frac{\rho'}{\rho} = -\frac{(1 + \omega)}{\omega} \frac{\tilde{a}'}{\tilde{a}}, \quad (30)$$

This can be integrated to obtain,

$$\rho \approx \tilde{a}^{-(1+\frac{1}{\omega})}, \quad (31)$$

We then find from (24) and (31), the expansion of the *bulk scale factor* goes as,

$$\tilde{a} \approx y^{\frac{2\omega}{(1+\omega)}}, \quad (32)$$

and for TLED case we obtain similar to 4D standard universe, i.e.,

$$\rho \approx \tilde{a}^{-3(1+\omega)}, \quad (33)$$

and

$$\tilde{a} \approx y^{\frac{2}{3(1+\omega)}}. \quad (34)$$

Although, the relations (33 , 34) are similar to (15 , 16), but for SLED case we obtain the new relations (31 , 32).

## 4 Accelerated Expansion and Contraction of the *bulk scale factor*

We shall now consider particular values of  $\omega$  to describe the physical significance of the new relations (31) and (32). For *dark energy fluid* with  $-1 < \omega < -\frac{1}{3}$ , we obtain,

$$\rho \approx \tilde{a}^{m'}, \quad (35)$$

and

$$\tilde{a} \approx y^{n'}. \quad (36)$$

where  $0 < m' < 2$  and  $-\infty < n' < -1$ . In this case, unlike the TLED case, the energy density of the bulk increase and we have contraction in the bulk with ( $a > 0$ ).

Also, for  $\omega = -1$  or vacuum, we have,

$$\tilde{a} \approx a^0 = cte. \quad (37)$$

In this case the energy density is independent of  $\tilde{a}$ , but we have anti- inflation ( exponentially contraction ) for the scale factor as,

$$\tilde{a} \approx y^{-\infty}. \quad (38)$$

This means that for SLED we have contraction of the *bulk scale factor* with ( $a > 0$ ). Thus, for both TLED and SLED, the scale factor vanishes if  $y \rightarrow 0$  and the density at that  $y$  becomes infinite. This point may be called *Big Bang* in the balk.

## 5 Discussions and Conclusions

We considered the warped metric in brane-world scenario as the modified form of 5D RW metric in higher dimensional cosmology, which admits both SLED and TLED. We solved the Einstein equation for this space-time and demonstrated that the evolution of the *bulk scale factor* and

density for SLED case is new. Also for dark energy fluid and vacuum fluid we have both of accelerated expansion and contraction in the bulk. This means two types of extra dimensions seems to be important in this scenario and play two different roles.

Moreover, the following results and points can be considered:

1. In this higher dimensional cosmology scenario, with incorporation of hidden extra dimensions and the visible space-time dimensions, the unification of gravitational force and gauge forces seems to be more possible.

2. In this scenario, for TLED we have an accelerated expansion of *bulk scale factor* and the strong gravitational force in the bulk is likely repulsive, but for SLED we have an accelerated contraction of the *bulk scale factor* (38) and the strong gravitational force in the bulk is likely attractive. Consequently, in the bulk, the strong gravitational force can be attractive as well as repulsive. Thus, the quantum effects of gravitation are of great importance in the bulk and in the higher dimensional cosmology [22].

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